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ON CONTROL OF THE PROCESS OF MATERIAL MACHINING LEADING TO REDUCTION IN THE RESIDUAL STRESSES*

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The problem of minimizing the level of residual stresses in a body in its natural state at the initial instant of time, in an inhomogeneous temperature field, is considered. A special function is obtained for estimating the level of residual stresses in terms of the running parameters of the process. An example is given of a solution of the problem of controlling the residual thermal stresses in an H-beam obtained by hot rolling.

Let the material occupying the region $\bar{D}$ under investigation be, at the initial instant $(\tau = 0)$ in its natural state (undeformed and stress-free). (We note that the initial deformations can be calculated without difficulty). During the period of cooling to ambient temperature $T_1 = \mathbf{const}\ (\mathbf{x})$ the material is subjected to various force and temperature influences some of which can be regarded as controls. The formation of residual stresses depends on the nonsteady transition from T_0 and T_1, and on the action of forces. We require to find the controls minimizing the residual stresses occurring when the region is cooled to the temperature T_1 of the surrounding medium. Separation of the forces into the phase variables and controls depends on the conditions of the problem in question. In many practical problems (in particular in the case quoted below) the forced local cooling or heating of some parts of the body surface can be used as the control. In this case the heat transfer coefficient as a function of time, and the coordinates, will serve as the control function.

The residual stresses ρ_{ij} satisfy the following system of equations (using the accepted notation):

$$\begin{aligned}
&\rho_{ij,j} = 0, \quad \mathbf{x} \in D \qquad (1)\\
&\rho_{ij} = C_{ijkl}(\varepsilon_{kl} - \varepsilon_{kl}'' - \varepsilon^T \delta_{kl}), \quad \mathrm{Ink}\varepsilon = \mathrm{rot}\,(\mathrm{rot}\,\varepsilon)^* = 0,\\
&\mathbf{x} \in \bar{D}\\
&\rho_{ij} n_j = 0, \ \mathbf{x} \in S\\
&\left(\varepsilon^T = \int_{T_0}^{T_1} \alpha_T(T)\,dT\right)
\end{aligned}$$

where $\alpha_T(T)$ is the linear thermal expansion coefficient. We assume here that the components ε_{ij} of the small residual deformation tensor ε can be written in the form of a sum of elastic ξ_{ij}, plastic ε_{ij} and temperature $\varepsilon^T\delta_{ij}$ terms. The asterisk denotes transposition and C_{ijkl} is the elastic constants tensor.

We see from (1) that the residual stresses can be affected only by using the plastic deformations ε_{ij}'', which can be found from the solution of the problem of thermoelastoplasticity corresponding to the cooling of the region from T_0 to T_1. In order to solve the problem of control of residual stresses, we must construct the corresponding special purpose function. In order to obtain solutions of the actual technical problems, the function must satisfy the following two conditions.

1°. The special purpose function must depend on the running parameters of the cooling process and not on the final parameters since in the latter case a repeated numerical solution of the associated nonlinear problem of thermoelastoplasticity, which is needed for the optimization, cannot in practice be obtained for the regions of complex form.

2°. In the real processes of hot working of metals the plastic deformations, and hence the residual stresses, can be acted upon only during the initial stage of the cooling process, as long as the yield limit remains sufficiently small. For this reason the special purpose function must recognise which plastic deformations should be carried out at the beginning of the cooling process in order to minimize the residual stresses at the end of the process.

Below we use the theory of Hilbert spaces /1,2/ to construct a special purpose function with the above-mentioned properties. It was shown in /3/ that the necessary and sufficient

*Prikl.Matem.Mekhan.,45,No.2,371-375,1981 condition for the absence of residual stresses within the region is the simultaneity of the elastic deformations at the instant when the region has been cooled to the ambient temperature T_1 and the force loading removed. It can therefore be expected that the residual stresses will depend on the degree of nonsimultaneity of the deformations.

If the residual elastic deformations ξ_{ij} are separated in some manner into the simultaneous $\xi_{ij}^{(1)}$ and nonsimultaneous $\xi_{ij}^{(2)}$ components, then it can be shown that the level of residual stresses is governed by the nonsimultaneous deformations $\xi_{ij}^{(2)}$ only. Indeed, from the first and fourth equation of (1) it follows that

$$\int_D \rho_{ij}\xi_{ij}\,dV = \int_D \rho_{ij}\xi_{ij}^{(2)}\,dV \tag{2}$$

and the Hooke's Law yields

$$\int_D \rho_{ij}\xi_{ij}\,dV \geqslant \frac{1-2\mu}{E}\int_D \rho_{ij}\xi_{ij}^{(2)}\,dV \tag{3}$$

Combining (2) and (3) we obtain

$$\int_D \rho_{ij}\rho_{ij}\,dV \leqslant \frac{E}{1-2\mu}\int_D \rho_{ij}\xi_{ij}^{(2)}\,dV \tag{4}$$

To obtain further estimates we introduce a Hilbert space H of symmetric bivalent tensors Here we encounter a number of possibilities, in particular we can introduce the energy space. Remembering however that the estimate (4) has a concrete form and that the orthonormed basis is constructed very simply, below we shall use the Hilbert spaces according to the formulas (5) and (6). Let us assume that the functions $W(x)$ are square summable, i.e.

$$\int_D W_{ij}W_{ij}\,dV < \infty \tag{5}$$

Then we can introduce a scalar product into the norm of the element W

$$(A, B) = \int_D A_{ij}B_{ij}\,dV, \quad \|W\| = (W, W)^{1/2} \tag{6}$$

Further we define in H a subspace H_1 of symmetric bivalent tensors the components of which satisfy the conditions of compatibility, i.e. have the form

$$\varepsilon_{ij} = {}^1/_2\,(u_{i,j} + u_{j,i}),\ \mathbf{x} \in \bar{D},\ \{\varepsilon_{ij}\} \in H_1$$

where u_i are the components of a vector. We note that the subspace H_1 is, generally speaking, not closed. This is due to the fact that a sequence of derivatives of continuous functions can converge to a function which will not be a derivative of any function whatsoever (the derivative is understood to be defined in the usual, and not in the generalized sense).

The degree of incompatibility of the tensor can be estimated from the distance in the space H between the given tensor and the subspace H_1. This distance is defined as follows:

$$\rho(W, H_1) = \inf\{\rho(W, Y):\ Y \in H_1\}$$
$$(\rho(W, Y) = \|W - Y\|)$$

Let us now apply the Cauchy–Buniakowski inequality

$$\int_D \rho_{ij}\xi_{ij}^{(2)}\,dV \leqslant \|P\|\,\|E^{(2)}\|$$
$$\|P\| = \Big(\int_D \rho_{ij}\rho_{ij}\,dV\Big)^{1/2}, \quad \|E^{(2)}\| = \Big(\int_D \xi_{ij}^{(2)}\xi_{ij}^{(2)}\,dV\Big)^{1/2}$$

Then from (4) we have

$$\|P\| \leqslant \frac{E}{1-2\mu}\,\|E^{(2)}\| \tag{7}$$

The last inequality holds for any $\xi_{ij}{}^{(2)} = \xi_{ij} - \xi_{ij}{}^{(1)}$ including the components of the tensor $\| E^{(2)} \|$ with the minimal norm. We know from the Beppo—Lewy theorem /1,2/ that the minimal norm of $\| E^{(2)} \|$ has the corresponding component $\xi_{ij}{}^{\perp}$ of the tensor $E^{\perp}$ orthogonal to the closure $\bar{H}_1$ of the subspace H_1

$$\xi_{ij}{}^{\perp} = \xi_{ij} - (\mathrm{pr}_{\bar{H}_1} E)_{ij}$$

$$\| E^{\perp} \| = \inf \| E^{(2)} \| = \inf \| E - E^{(1)} \| = \| E - \mathrm{pr}_{\bar{H}_1} E \|$$

where $(\mathrm{pr}_{\bar{H}_1} E)_{ij}$ are the components of the projection of the tensor E, with components ξ_{ij}, on the subspace H_1.

Let us introduce the notation

$$r_{ij}(\tau) = \varepsilon_{ij}{}^{p}(\tau) + \varepsilon^{T}\delta_{ij}, \ \mathbf{x} \in \bar{D}, \ \{r_{ij}\} = R$$

where $r_{ij}(\tau)$ is the sum of the running plastic $\varepsilon_{ij}{}^{p}(\tau)$ and total temperature $\varepsilon^{T}\delta_{ij}$ components of the deformation tensor. Using the simultaneity of the total deformations it can be shown that $\xi_{ij}{}^{\perp}$ coincide, with the accuracy of up to the sign, with the components $r_{ij}{}^{\perp}(\tau_1)$ of the tensor $R^{\perp}(\tau_1)$ (here τ_1 denotes the instant at which the region attains the temperature T_1).

If the orthonormed basis $\Pi^{(n)}$ in the subspace H_1 is known (it also serves as the basis in $\bar{H}_1$), then

$$[\mathrm{pr}_{\bar{H}_1} R(\tau_1)]_{ij} = \sum_{n=1}^{\infty} \alpha_n \pi_{ij}^{(n)}, \quad \alpha_n = \int_{D} r_{ij}(\tau_1)\, \pi_{ij}^{(n)}\, dV$$

is the Fourier coefficient on the basis $\Pi^{(n)}$. By definition of the projection, we have

$$\| E^{\perp} \| = \| R^{\perp}(\tau_1) \| \leqslant \| R(\tau_1) - R^{(1N)}(\tau_1) \|$$

where the components $r_{ij}^{(1N)}(\tau_1)$ of the tensor $R^{(1N)}(\tau_1)$ are given by

$$r_{ij}^{(1N)}(\tau_1) = \sum_{n=1}^{\infty} \alpha_n \pi_{ij}^{(n)}.$$

As a result, the inequality (7) yields the final estimate of the level of residual stresses

$$\| P \| \leqslant \Phi[\varepsilon_{ij}^{p}(\tau_1)] = \frac{E}{1-2\mu} \| R(\tau_1) - R^{(1N)}(\tau_1) \| = \frac{E}{1-2\mu} \left[\| R(\tau_1) \|^2 - \sum_{n=1}^{N} \alpha_n^2 \right]^{1/2} \tag{8}$$

In order to use the estimate (8), we must construct the basis $\pi_{ij}^{(n)}$ in the subspace of simultaneous deformations. This can be done with help of the displacement fields. The functional introduced in /4/ follows from (8), with the basis tensors used as constants.

We note that the functional $\Phi[\varepsilon_{ij}{}^{p}(\tau_1)]$ can be computed at the instant τ_k after which no plastic deformations occur at any point of the region in question. We illustrate the method of solving an actual problem by considering the problem of minimizing the residual thermal stresses in an H-beam 60Sh1 (wide flange beam with 60 cm distance between the flanges) made of steel St.20. We assume that after rolling the beam is stress-free and that the temperature distribution across the transverse section $T_0(\mathbf{x})$ is known for the solution of the problem of heat conduction during the rolling process. The residual stresses were found from the solution of the problem of thermoelastoplasticity corresponding to the cooling of the beam on a cooling rig. All computations were carried out using the finite element method, and the solution is described in greater detail in /4/. The local forced cooling of the external flange surfaces was used as the control. The search for the minimum of the functional $\Phi(\tau)$ (8) was carried out using the method of deformable polyhedron.

We note that the characteristic feature of the profiles obtained by hot rolling is, that the stress along the profile axis is much greater than the remaining components of the stress tensor. In this case the estimate (8), as was shown in /4/, becomes an equality. The minimization of the functional $\Phi(\tau)$ at every step is equivalent, with respect to time, to the process of obtaining a deformed state, with the deformations $\varepsilon_{ij}{}^{p}(\tau) + \varepsilon^{T}\delta_{ij}$ approaching closer and closer the compatible deformations. This is accompanied by decrease in the potential energy of the residual stresses.

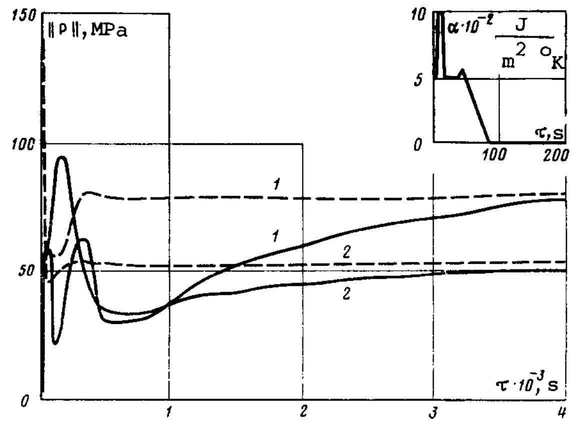

Fig.1

The Fig.1 depicts how the heat transfer coefficient α between the coolant and the metal varies with time τ under the optimal conditions. Estimates of the level of internal stresses are also compared, the solid lines *1* and *2* giving the running values under the spontaneous cooling and in the optimal regime of forced cooling and the dashed lines *1* and *2* the corresponding values obtained from the functional $\Phi(\tau)$ (8). The peaks appearing at small values of τ are caused by the phase transitions within the profile.

The solution has shown that the maximum longitudinal compressive stresses in the middle of the wall are reduced from −80 MPa (under the normal cooling) to −25 MPa (under the optimal cooling regime). The maximum tensile stresses in the region between the wall and the flange fell from +160 MPa to +80 MPa.

The total time taken to solve the problem of optimization on BESM-6 was 2.5 hours.

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